Addition Strategies Continuum

Count All A

4 + 3 =

Count 4, then count 3, then count all.

1, 2, 3, 4

O O O O

1, 2, 3

O O O

1, 2, 3, 4, 5, 6, 7

O O O O O O O

Count On A

4 + 3 =

4 + /// = 7

Count or say 4, then continue counting 5, 6, and 7 without going back to 1.

Doubles Plus or Minus A

6 + 7 =

6 + 1

6 + **6** + 1 = 13
(or **7** + **7** – 1 = 13)

Doubles are easier facts to recall and can be very useful.

Working With Fives A

6 + 7 =

5 + 1 + **5** + 2 =

10 + 3 = 13

Decompose numbers to identify fives within numbers and add.

Making Tens A

9 + 7 =

9 + 1 6 + **1**

10 + 6 = 16

Identify tens within numbers and add.

Using Compensation A

8 + 5 =

3 + **2**

8 + 2

10 + 3 = 13

Compensate by shifting quantities from one addend to the other.

Using Known Facts A

7 + 8 =

Think about the numbers and how they may be related.

I know that 6 + 8 = 14. So ...

7 + 8
must be 15.

Splitting A

27 + **4**4 =

20 + 7 **40** + 4

20 + **40** = 60

7 + 4 = 11

60 + 10 + 1 = 71

This is a landmark strategy using an understanding of place value.

Jumps of 10 A

27 + 44 =

44 + 10 + 10 + 7 = 71

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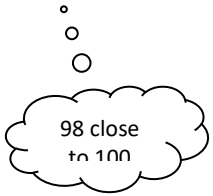
348 + 159 =

348 + 100 + 50 + 9 = 507

One number is kept whole and multiples of 10 are added.

Next Friendly Number A

98 + 37 =



100 + 35 = 135

Compensate by shifting quantities to reach a "friendly" number and adjust other addend accordingly.

Points to Consider A

- The underlying concept of addition is embedded in counting.
- Students move from counting to combining collections.
- Strategies that require understanding of the commutative and associative properties of addition begin to develop.
- Addition or subtraction can be used to solve missing-addend problems successfully.
- Students develop the concept of inverse operations as they continue to work with addition and subtraction.

Key – Strategies Continuum

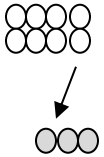
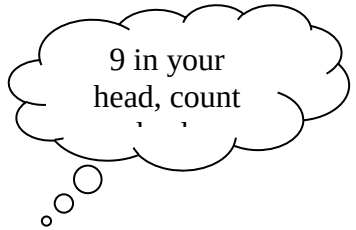
A **Addition**

S **Subtraction**

M **Multiplication**

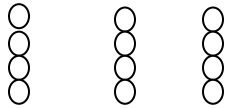
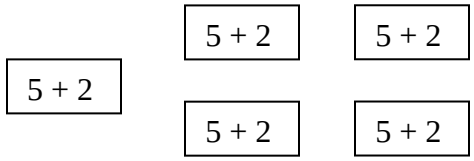
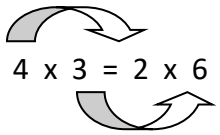
D **Division**

Subtraction Strategies Continuum

Using Counters	s	***Counting Up***	s	***Counting Back***	s
8 - 5 = Count out 8 chips and remove 3 chips.  There are 5 chips left. 		8 - 5 =  5, 6, 7, 8 		9 - 2 =  9, 8, 7 	
Making a 10	s	***Jumps of 10***	s	***Using Doubles***	s
Knowing 6 + 4 = 10 can be helpful in finding differences from 10 ... 10 – 6 = 4 and 10 – 4 = 6 May be used to reinforce close facts: 9 – 4 = 5 11 – 4 = 7		Using a pattern of jumps of 10 ... 17 – 10 = 7 13 – 10 = 3 ...may be used to reinforce close facts: 17 – 9 will be 1 more than 17 – 10.		Knowing 8 + 8 = 16 and 6 + 6 = 12 Uses the idea of “half doubles”: 16 – 8 = 8 12 – 6 = 6 <u>May be used</u> to reinforce close facts: 15 – 8 = 7 13 – 6 = 7	

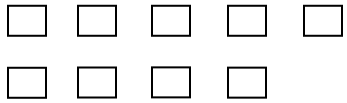
Related Addition Facts	**S**	***Points to Consider***	**S**	***Points to Consider***	**S**
Use reasoning to draw from known facts. Knowing that 8 + 7 = 15 is helpful in solving 15 – 7 = 8. Seeing relationships between operations is vital for fluency and flexibility.		• Subtraction and addition are inherently connected, so strategies are similar. • An understanding of the relationship between addition and subtraction needs to be developed within contexts first. • "Adding on" versus "removing" is a central idea in subtraction; it is important to know when to use which strategy. When numbers are closer together, it is easier to add on; when they are farther apart, it is easier to work backward.		• Students need to use and understand a variety of vocabulary that describes subtraction. "Take away" implies a strategy, and if it is the only vocabulary used, it determines the strategy. Other vocabulary words that can be used include difference, subtract, and remove. • Compensation is a common strategy for addition, and many students will generalize this idea and apply it to subtraction. However, it does not work for subtraction. Subtraction requires that a "constant difference" remain. • Students use many models to show their thinking. Models may include number lines, splitting, swapping, friendly numbers, cancelling out common amounts, etc. All are ways to demonstrate understanding.	

Multiplication Strategies Continuum

Representing Each Item Counting by Ones M	***Repeated Addition (Skip Counting)*** M	***Unitizing (Counting By)*** M
How many crackers are there total in 3 bags containing 4 crackers each? Count each group separately, then count the whole. 	How many crackers are there total in 3 bags containing 4 crackers each? Count by saying 4 + 4 = 8 and 8 + 4 = 12 or by saying 4, 8, 12. 	How many toy cars are there total in 5 boxes containing 7 cars each?  Five 5s and five 2s: 5, 10, 15, 20, 25, plus 2 x 5, which equals 10. 25 + 10 = 35
Doubling M	***Halving and Doubling*** M	***Using the Distributive Property*** M
2 x 3 x 6 = 6 x 6 Recognize that 2 x 3 is a double of 3, which is 6. Support flexibility with number sense.	 Split the 4 in half and double the 3. Supports flexibility with number sense.	9 x 6 (5 x 6) + (4 x 6) 12 x 11 (6 x 11) + (6 x 11) or (12 x 12) - 12

Using the Distributive Property with Tens M	***Using the Commutative Property*** M	***Associative Property*** M
9 x 8 (10 x 8) - 8 23 x 19 (23 x 20) - 23	6 x 7 = 7 x 6 28 x 5 = 5 x 28 30 x 12 = 12 x 30	(7 x 5) x 3 = 7 x (5 x 3) (The order in which the factors are multiplied does not affect the answer.)
Points to Consider M	***Points to Consider*** M	
• Skip counting is helpful in learning multiplication facts, as is the array model. • Students should be exposed to many different types of problems and situations involving multiplication. • Repeated addition is an effective model for multiplication of whole numbers.	• Multiplication does not always result in a larger product. • Multiplication and division are inverse operations. • Understanding properties and their role in the operation supports number sense.	

Division Strategies Continuum

Representing Each Group and Counting All (Quotative Situation) D	***Repeating Addition, Skip Counting, Counting On (Quotative Situation)*** D	***Using the Descriptive Property of Multiplication (Quotative Situation)*** D
How many tables will we need for 36 people if we seat 4 people at each table?  **1, 2, 3, 4, 5, 6, 7 ... 36** Make representations of the tables and add 4 tallies for each table, then count tallies up to 36. Now go back and count the number of tables the 36 tallies occupy.	How many tables will we need for 36 people if we seat 4 people at each table? 4 4 4 4, 8, 12, 16 ... 36 Make representations of the tables and put a 4 on each table. Count by 4s to 36 and determine the number of tables by the number of 4s said.	How many tables will we need for 36 people if we seat 4 people at each table? 5 x 4 = 20 4 x 4 = 16 9 x 4 = 36 Use simpler multiplication facts to determine the number of 4s needed to reach 36 (9 tables needed).
Using the Commutative Property of Multiplication (Paritive Situation) D	***Points to Consider*** D	***Points to Consider*** D
If there are 36 people to seat at 9 tables, how many can sit at each table? Count out 36 cubes and put them one by one into 9 groups. Stop when all cubes are distributed and count the number of cubes in one of the groups. This strategy and paritive situation relies on a student's knowledge of fair shares and equal groups.	• Division is more than repeated subtraction or the opposite of multiplication. • There are two types of division context students need to be exposed to : quotative and paritive. Each kind of problem is solved using a different strategy. • Quotative situations asks how many groups there are when the size of the group is known.	• Paritive situations ask how many are in the group when the number of groups is known. • Division is central to working with fractions, percents, and proportions. • Knowing multiplication facts and understanding multiplicative situations is essential to success in division.